Weird light propagation

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Outline

• Accelerating diffractionless light beams?

• Airy wave-packets in quantum mechanics and optics

• Other accelerating beams: Mathieu, Weber, Fresnel...

• Accelerating beams in photonic crystals

• Beams in fractional Schrödinger equation
Light travels in straight lines - Right?

Courtesy of D. Christodoulides

Euclid of Alexandria
325-265 BC

Euclid’s Optics
~ 300 BC

Yes, but...
In arts – yes!
In physics?
But, can light accelerate?

Sure!

CREOL - The College of Optics and Photonics
How about **Diffraction-free patterns?**

Simple pattern: Two-wave mixing

\[
E = E_0 \exp[i(k_x x + k_z z)] + E_0 \exp[i(-k_x x + k_z z)] \\
E = 2E_0 \cos(k_x x) \exp[i k_z z] \\
I = |E|^2 = 4E_0^2 \cos^2(k_x x)
\]

**Propagating diffraction-free waves often accelerate!**
Non-diffracting beams - conical plane wave superposition

4-waves

21 waves
It started all in quantum mechanics: Airy wave

M.V. Berry and N. L. Balazs, “Nonspreading wave packets,”
Am. J. Phys. 47, 264 (1979)

Free-particle Schrödinger equation

\[
i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = 0
\]

Unique Airy wave-packet solution

\[
\psi(x,t) = Ai \left[ \frac{B}{\hbar^{2/3}} \left( x - \frac{B^3 t^2}{4m^2} \right) \right] e^{iB^3 t/2m \hbar} [x-(B^3 t^2/6m^2)].
\]

Non-spreading Airy wave-packet

|Ψ|^2
\[
t > 0 \quad \rightarrow \quad \text{acceleration}
\]

Where from?

Courtesy of Ady Arie
The Airy function is named after the British astronomer Airy, who introduced it during his studies of rainbows.

Equation:
\[ y''(x) = xy \]

Solution:
\[ y(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ipx} e^{ip^3/3} \, dp \equiv Ai(x) \]

How come?
Solution? Go to inverse space, young man

- Airy equation
  \[ \frac{d^2 y}{dx^2} = xy \]
- Fourier transform
  \( \hat{y}(p) = \int_{-\infty}^{\infty} e^{-ipx} y(x) \, dx \)
- Derivatives
  \[ \frac{d}{dx} \leftrightarrow +ip, \quad \frac{d}{dp} \leftrightarrow -ix \]
- Equation in the inverse space
  \( -p^2 \hat{y} = i \frac{d\hat{y}}{dp} \)
- Solution
  \[ \hat{y}(p) = e^{ip^3/3} \quad y(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ipx} e^{ip^3/3} \, dp \equiv Ai(x) \]

- Paraxial wave equation
  \[ i\partial_z u(\rho, z) = -\Delta u(\rho, z) \]
  \[ \frac{\partial u}{\partial x} \rightarrow ik_x \tilde{u} \quad \text{grad} u \rightarrow i\tilde{q} \tilde{u} \quad \Delta u \rightarrow -q^2 \tilde{u} \]
  \[ i \frac{\partial \tilde{u}}{\partial z} = q^2 \tilde{u} \Rightarrow \tilde{u} = e^{-i\rho^2 z} \tilde{u}_0 \quad u(z, \rho) = (FT)^{-1} \exp(-iq^2 z)(FT)u(0, \rho) \]
From Quantum Mechanics to Optics

\[
\begin{align*}
&i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = 0 \\
&i \frac{\partial \Phi}{\partial \xi} + \frac{1}{2} \frac{\partial^2 \Phi}{\partial s^2} = 0
\end{align*}
\]

Free particle
Schrödinger equation

Infinite energy
wave packet

- Non diffracting
- Freely accelerating

Berry and Balzas, 1979

Scaled paraxial wave
equation

Finite energy beam

\[ Ai(s)e^{as} \]

Siviloglou and Christodoulides, 2007

- Nearly non diffracting
- Freely accelerating

Nondiffracting optical waves

2D

Airy beam in 1D

\[ \psi(s, \xi) = A i (s - \frac{\xi^2}{4}) \exp\left[i \left(\frac{s \xi}{2} - \frac{\xi^3}{12}\right)\right] \]

Q: What is accelerating?

Free fall

- The only possible nondiffracting wave in 1D
- Self-healing property
- Transverse momentum (self-bending)

Courtesy of Z. Chen
In optics, Airy beam is a manifestation of caustic.

**Caustic:** An envelope of light rays reflected or refracted by a curved surface or object or the projection of that envelope of rays on another surface.

In a ray description, the rays are tangent to the parabolic line but do not cross it.

The Airy beam is a beam with curving propagation trajectories of its lobes.

Curved caustics in everyday life

Caustics are everywhere

Pool caustics

Nephroid caustics
Still: Optical analog of projectile ballistics

The Airy beam moves on a parabolic trajectory very much like a projectile under the action of gravity!

It looks like
But it’s not
What it looks like
Applications of Airy beams

Curved plasma channel generation in air

Transporting micro-particles

Airy–Bessel wave packets as versatile linear light bullets in 3D

Micromachining using accelerating beams.
Airy beams propagating in NL media

- Equation

\[ i \frac{\partial \psi}{\partial z} - i \frac{z}{2} \frac{\partial^2 \psi}{\partial x^2} + \frac{1}{2} \frac{\partial^2 \psi}{\partial x^2} + \delta n \psi = 0. \]

\[ \psi(x) = A_1 \text{Ai}[(x - B)] \exp[a(x - B)] + \exp(il\pi) A_2 \text{Ai}[-(x + B)] \exp[-a(x + B)]. \]

- Input beam

Single Beam Prop

No truncation, \( a = 0 \)

Truncated \( a = 0.2 \)
Counter-accelerating NL Airy beams

- Kerr medium: Generation of solitons
- No acceleration!
- Upper row: In-phase (attraction)
- Lower row: Out-of-phase (repulsion)

Paraxial accelerating beams; 2D

- Paraxial wave equation

\[
\nabla^2 \psi + i \partial_s \psi = 0 \quad s = \frac{z}{2k\kappa^2}
\]

- Solution

\[
\psi(u,v,s) = e^{-isp^2} \psi(u,v,0) \quad p^2 = -\nabla^2
\]

- 1D case: Usual parabolic Airy beam

- 2D case:

\[
e^{-isp^2} = e^{-i2s^3/3} e^{isu} e^{-is^2 p_e} e^{-is\hat{H}}, \quad \hat{H} = - (\partial_u^2 + \partial_v^2) + u, \quad p_u = -i \partial_u
\]

- Initial eigenvalue problem:

\[
\hat{H} \psi(u,v,0) = \lambda \psi(u,v,0)
\]

\[
\psi_0(u,v) = \frac{1}{2\pi} \int e^{ik \cdot \rho} \tilde{F}_0(k_u,k_v) dk, \quad (k_u^2 + k_v^2 + i\partial_{k_u}) \tilde{F}_0(k_u,k_v) = 0
\]

\[
\psi(u,v,s) = e^{isu} e^{-is^2} e^{-is^3/3} \times \quad \tilde{F}_0(u - \lambda - s^2, v)
\]

M. Bandres: *OL* 34, 3791 (2009)
Nonparaxial accelerating beams; 3D

- Helmholtz equation
  \[(\partial_{xx} + \partial_{yy} + \partial_{zz} + k^2) \psi = 0\]

- Solution
  \[\psi(r) = \int A(\theta, \phi) \exp(i k r \cdot u) \, d\Omega,\]

- \(A(\theta, \phi)\) Angular spectrum function
  \[u = (\sin \theta \sin \phi, \cos \theta, \sin \theta \cos \phi) \quad d\Omega = \sin \theta \, d\theta \, d\phi\]

- Assume \(A(\theta, \phi) = g(\theta) \exp(im \phi)\)

\[
\psi(r) = \int_0^\pi \int_{-\pi/2}^{\pi/2} g(\theta) \exp(im \phi) \exp(i k r \cdot u) \sin \theta \, d\theta \, d\phi,
\]

- For different beams pick different spectral functions

Interesting cases with separable Helmholtz equation
Coordinate systems in which the Helmholtz equation is separable

- Parabolic acc beam (AB)
- Spheroidal AB
  - Oblate
  - Prolate
- Spherical
Nonparaxial accelerating beams

Parabolic

Prolate Spheroidal ABs

Oblate Spheroidal ABs

2-D

Half-Bessel

Weber

Half-Mathieu

Parabolic

Oblate

Spherical

Prolate

3-D
Accelerating beams in photonic crystals

Beams in periodic systems: Energy band structure!

Highly nonparaxial: Helmholtz eq. $E_{xx} + E_{zz} + k_0^2 \varepsilon(x, z) E = 0$

Inherently counterprop: Forward and backward Bloch modes must be present:

$$u_{k_0, k, k_z}(x, z) e^{i k \cdot r} \quad k = (k_x, k_z)$$

$$E_{k_0} ((x, z) = r) = \oint_{k \in \text{Curve}(k_0)} w(k) u_{k_0, k}(r) e^{i k \cdot r} \, dk$$

Courtesy of M. Segev
Fig. 2. Examples of self-accelerating beams in a 2D photonic crystal slab. For each solution, we plot the amplitude (top row) and phase (bottom row). The black arrows schematically mark the Poynting vector hence the energy flow. (a) The full “whirlpool beam”, completing a full circle, is created from two counter-propagating input beams launched from top and bottom of the photonic crystal slab. (b) The self-accelerating beam initiated from the bottom of the slab.

Halo?
Dynamics of beams in Fractional SE

- Fascinating new field, “FQM” by Laskin, born in 21st century
- Longhi: “Fractional Schrödinger equation in optics”

\[ i \frac{\partial F}{\partial t} + \frac{1}{2} \left( -\frac{\partial^2}{\partial x^2} \right)^{\alpha/2} F + V(x) F = 0 \]

- NL:

\[ i \frac{\partial F}{\partial z} + \frac{1}{2} \left( -\frac{\partial^2}{\partial x^2} \right)^{\alpha/2} F + |F|^2 F + V(x) F = 0 \]

- The fractional Laplacian of order \(1 < \alpha < 2\) is defined by:

\[ \left( -\frac{\partial^2}{\partial x^2} \right)^{\alpha/2} F(x) = \frac{1}{2\pi} \int \int_{-\infty}^{\infty} dp d\xi |p|^\alpha F(\xi) \exp[ip(x - \xi)]. \]

Go to inverse space, young man!
Representative results: Linear, $\alpha=1$

- Harmonic potential

$$i \frac{\partial F}{\partial t} + \frac{1}{2} \left( - \frac{\partial^2}{\partial x^2} \right)^{1/2} F + \frac{x^2}{2} F = 0$$

- Equation in $k$-space

$$i \frac{\partial \hat{\psi}}{\partial \xi} - \left( \frac{1}{2} |k| - \frac{1}{2} \frac{\partial^2}{\partial k^2} \right) \hat{\psi} = 0$$

$$\psi(x) = \exp \left[ -\sigma (x - x_0)^2 \right]$$

- Input Gaussian beam

- Propagation of

- Displaced Gaussian

Dashed curves: Analytical
Shaded regions: Numerical

Y. Zhang et al., PRL 115, 180403 (2015)
Representative results: Solitons in NL FSE

- Kerr nonlinearity

\[ i \frac{\partial F}{\partial z} + \frac{1}{2} \left( - \frac{\partial^2}{\partial x^2} \right)^{\alpha/2} F + |F|^2 F + R(x) F = 0 \]

\[ R(x) = p[1 - \cos(\Omega x)] \]

- Potential: Ref. index modulation in form of a lattice boundary

- Assumed solution

\[ F(x, z) = w(x) \exp(ibz) \]

- Equation

\[ \frac{1}{2} \left( - \frac{\partial^2}{\partial x^2} \right)^{\alpha/2} w + |w|^2 w - Rw + bw = 0 \]

- Formation of surface solitons at the boundary

Stable Unstable →
Summary

Presented nondiffracting Airy wave-packets and beams

Airy (and other) wave-packets are freely accelerating and shape preserving

In optics it is a curving beam following parabolic trajectory

Presented various nonparaxial nondiffracting accelerating beams: Mathieu, Weber, Fresnel, Photonic, Fractional

Alluded on ways to introduce nonlinearities and applications

Thank you for your attention!